SNR Maximization Based on Joint Power Control and Beamforming for AF Relay Network

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Abstract

In this paper, an one-source one-destination system is considered which uses multi single antenna amplify and forward (AF) relays to connect end nodes. By ignoring the effect of direct link between source and destination nodes due to the high path loss, we find a beamforming regime at relays such that the achieved signal to noise ratio (SNR) is maximized and the total relay transmit power is set lower than a certain threshold. We show that the problem of finding beamforming matrix is a non-convex problem. So, we use semi-definite relaxation technique to convert it to a semi definite programming (SDP) problem. Then, we propose three methods for maximizing the achieved SNR. Finally, the perfomance of the proposed methods is compared and the effects of the number of relays and the qualities of channels are evaluated by simulation, numerically.

Keywords

Power Control; Relay Beamforming; MIMO; AF Relay

Introduction

The growth of wireless networks in recent years is motivated by their ability of providing communications anywhere and anytime. Because of the importance of this aspect on the modern society, a high proliferation of wireless services and devices such as mobile communications, WiFi (Wireless Fidelity) or cordless phones has emerged. However, in contrast to the wired networks, the wireless networks present two main drawbacks: the scarcity of radio spectrum and channel impairments. Therefore, the wireless networks should be designed to exhibit a high spectral efficiency and combat channel impairments including multipath fading, shadowing, interference and path-loss for an enhanced homogeneous coverage.

Recently, advances in radio transceiver techniques such as Multiple Input Multiple Output (MIMO) architectures have shown an enhancement in the capacity of the current systems by dealing with the channel multipath fading. This is possible by adding multiple antennas at the transmitter and/or the receiver. Another technique which has recently gained attention is the cooperative communication.

The cooperative transmission is a relatively new class of spatial diversity technique where a new element comes up in the conventional source-destination or point-to-point communication. The relay assists the source in transmitting a message to the destination and allows dealing with the channel impairments. In fact, the relay-assisted transmission can be seen as a virtual MIMO with distributed antennas. In contrast to MIMO systems, the transmission requires the use of additional channel resources because of the limitation of the current radio technology: the relay terminal is constrained to work in half-duplex mode, which motivates that the transmission must be carried out in two orthogonal phases (relay-receive and relay-transmit phases), and duplexed in time or frequency domains.

There are various cooperative schemes such as Amplify and Forward (AF) [Khajehnouri et al., 2007], Decode and Forward (DF), [Laneman et al., 2004] etc. Between these several schemes, AF relaying is the most practical interest, because of its simplicity. Distributed relay beamforming has tremendous capability to power saving in relays using adjustable transmit power and is well investigated in the literature with different design issues and channel information assumptions [Zhang et al., 2009], [Havary-Nassab et al., 2008], [Havary-Nassab et al., 2010]. In [Zhang et al., 2009], joint beamforming and power control for multi-antenna relay broadcast channel was investigated with the full Channel State Information (CSI) assumption. When there is no CSI in the

transmitter, space-time coding can achieve full diversity [Jing et al., 2006]. In [Havary-Nassab et al., 2008], two multi-relay distributed diagonal full rank beamforming technique was developed based on two different optimality criteria, namely total relay transmit power minimization under quality of service constraint and Signal to Noise Ratio (SNR) maximization under a total relay power budget constraint and individual relay power constraint with the second-order statistics of CSI assumption. In [Havary-Nassab et al., 2010] the problem of SNR maximization under a total relay power constraint using general-rank complex matrix has been well investigated.

In this paper, we propose a new beamforming approach to maximize SNR subject to relay power constraint. To do so, in the first phase, source transmits its signal to the relays. Then, the relays manipulate the received signal with the complex beamforming matrix and retransmit it. Our goal is to obtain complex beamforming matrix of the relay as the design parameter, such that the SNR is maximized. Study shows that this problem is a non-convex problem. Therefore, we use a semi-definite relaxation technique to convert the problem into a Semi-Definite Programming (SDP) problem. We test the effect of the number of relays and quality of channels on the proposed algorithm, numerically.

System Model

A cooperative system is considered where a single-antenna source transmits its signal to a single-antenna destination while multi single-antenna AF relay supports this transmission, as illustrated in Fig. 1.

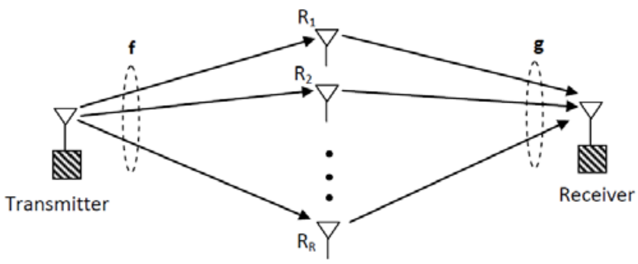

FIG. 1 A SYSTEM CONSISTING OF SINGLE-ANTENNA SOURCE-DESTINATION AND MULTI SINGLE-ANTENNA AF RELAYS

Due to path-loss, it is assumed that there is no direct link between the source and the destination node, similar to [Khandaker et al., 2012], [Fazeli-Dehkordy et al., 2009]. So, the source can communicate with the destination only by the relay node.

We denote the Rayleigh flat fading channel vectors between the source and the relay nodes and between the relay and the destination nodes by $\mathbf{f} = [f_1 f_2 \dots f_R]^T$, and $\mathbf{g} = [g_1 g_2 \dots g_R]^T$, respectively.

Also, the correlation matrices of the channel vectors $\mathbf{f}$ and $\mathbf{g}$ can be expressed as $\mathbf{R}_f = E\left\{\mathbf{ff}^H\right\}$ and $\mathbf{R}_g = E\left\{\mathbf{gg}^H\right\}$, respectively.

Let s be the information symbol transmitted by source, P_0 be the transmit power and v_r be the additive zero-mean noise at the rth relay node. We assume that $E\left\{|s|^2\right\} = 1$. Also, we assumed the relay noise is spatially white, i.e., $E\{v_r v_{r'}^*\} = \sigma_v^2 \delta_{rr'}$ where σ_v^2 is the relay noise power. Moreover, it is assumed that the information symbols and the rth relay noise v_r are statistically independent, all nodes (the source, the destination and the relay ones) are half-duplex so that cooperative transmission must be conducted over two phases. In phase I, the source first transmits the signal to the relay node. In this way, relay received signal x_r is

$$x_r = \sqrt{P_0}\, f_r s + v_r \tag{1}$$

In phase II, the relay multiplies its signal vector received by beamforming weight coefficients, w_r^*, and then forwards it to the destination. The vector of signal transmitted by relay is

$$t_r = w_r \; x_r \tag{2}$$

and the destination received signal can be given by:

$$y = \sum_{r=1}^{R} g_r t_r + n \tag{3}$$

Using (1) and (2), we can rewrite (3) as:

$$y = \sum_{r=1}^{R} g_r w_r x_r + n = \underbrace{\sqrt{P_0} \sum_{r=1}^{R} g_r w_r f_r s}_{desired\ signal} + \underbrace{\sum_{r=1}^{R} g_r w_r v_r + n}_{noise} \tag{4}$$

where n is the zero-mean noise at destination with a variance of σ_n^2.

Snr Maximization

Our goal is to find the complex beamforming weights $\left\{w_r\right\}_{r=1}^{R}$ that maximize SNR while the transmit power is kept below a pre-defined value. In other words, we aim to solve the following optimization problem:

$$\begin{aligned} &\max && SNR \\ &subject\ \ to && P_t \le P_t^{Max} \end{aligned} \tag{5}$$

where P_t is the total transmit power dissipated at the relay nodes, P_T^{Max} is the maximum allowable total transmit power and SNR is defined as:

$$SNR = \frac{P_S}{P_n} \tag{6}$$

where P_S and P_n are the desired signal and the total noise powers at the destination, respectively. The total transmit power at the relay nodes can be obtained as:

$$P_T = \sum_{r=1}^{R} E\left\{|t_r|^2\right\} = \sum_{r=1}^{R} |w_r|^2 E\left\{|x_r|^2\right\} = \mathbf{w}^H \mathbf{D}\mathbf{w} \tag{7}$$

Where $\mathbf{D} = P_0 diag\left(E\left\{|f_1|^2\right\} \quad E\left\{|f_2|^2\right\} \quad \dots \quad E\left\{|f_R|^2\right\}\right) + \sigma_v^2 \mathbf{I}$ and $\mathbf{w} = \left[w_1 \quad w_2 \quad \dots \quad w_R\right]^T$.

Considering the assumptions on the previous works, assuming that the channel coefficients, the relay noises and the receiver noise n are all independent from each other, using (4) the total noise power at receiver can be written as:

$$P_n = E\left\{\sum_{m,n=1}^{R} w_m w_n^* g_m g_n^*\right\}\sigma_v^2 + \sigma_n^2 = \mathbf{w}^H \mathbf{Q}\mathbf{w} + \sigma_n^2 \tag{8}$$

where $\mathbf{Q} = \sigma_v^2 E\left\{\mathbf{g}\mathbf{g}^H\right\}$ and $\mathbf{g} = \left[g_1 \quad g_2 \quad \dots \quad g_R\right]^T$.

The received signal power at the destination can be obtained as:

$$\begin{aligned} P_s &= P_0 E\left\{\sum_{m,n=1}^{R} w_m w_n^* f_m g_m f_n^* g_n^*\right\} E\left\{|s|^2\right\} \\ &= P_0 \mathbf{w} E\left\{\mathbf{h}\mathbf{h}^H\right\}\mathbf{w} = \mathbf{w}^H \mathbf{R}\mathbf{w} \end{aligned} \tag{9}$$

where $\mathbf{h} = \left[f_1 g_1 \quad f_2 g_2 \quad \dots \quad f_R g_R\right]^T$ and $\mathbf{R} = E\left\{\mathbf{h}\mathbf{h}^H\right\}$.

Therefore by using (8) and (9) the SNR can be expressed as:

$$SNR = \frac{\mathbf{w}^H \mathbf{R}\mathbf{w}}{\mathbf{w}^H \mathbf{Q}\mathbf{w} + \sigma_n^2} \tag{10}$$

Now, we can rewrite the optimization problem (5) as:

$$\begin{aligned} &\max && \frac{\mathbf{w}^H \mathbf{R}\mathbf{w}}{\mathbf{w}^H \mathbf{Q}\mathbf{w} + \sigma_n^2} \\ &subject\ \ to && \mathbf{w}^H \mathbf{D}\mathbf{w} \le P_t^{Max} \end{aligned} \tag{11}$$

The problem in (11) is not a convex optimization problem and may not have a solution with affordable computational complexity. We exploit a semi-definite relaxation approach to solve a relaxed version of (11). To do so, we define $\mathbf{Z} = \mathbf{w}\mathbf{w}^H$. Therefore, the optimization problem in (11) can be rewritten as:

$$\begin{aligned} &\max && tr(\mathbf{A}\mathbf{Z}) \\ &subject\ \ to && tr(\mathbf{D}\mathbf{Z}) \le P_t^{Max}, rank(\mathbf{Z}) = 1, \mathbf{Z} \ge 0 \end{aligned} \tag{12}$$

Maximization of SNR can be achieved by one of the following proposed ideas which are named as algorithm 1, 2 and 3, respectively.

1) Minimizing the denominator: $\mathbf{A} = \mathbf{w}^H \mathbf{Q}\mathbf{w}$
2) Maximizing the numerator $\mathbf{A} = \mathbf{w}^H \mathbf{R}\mathbf{w}$
3) Maximizing the difference between the numerator and the denominator: $\mathbf{A} = \mathbf{w}^H \mathbf{R}\mathbf{w} - \mathbf{w}^H \mathbf{Q}\mathbf{w} + \sigma_n^2$

As the next problem, the rank constraint in (12) is not convex. Using semi-definite relaxation, we remove this non-convex constraint and solve the following optimization problem:

$$\begin{aligned} &\max && tr(\mathbf{A}\mathbf{Z}) \\ &subject\ \ to && tr(\mathbf{D}\mathbf{Z}) \le P_t^{Max}, \mathbf{Z} \ge 0 \end{aligned} \tag{13}$$

The optimization problem (13) is indeed convex and can be solved efficiently by using interior point based software tools such as CVX [Grant et al., 2011]. But due to the relaxation, the matrix Z obtained by solving the optimization problem in (13) will not necessarily be of rank one. As it is shown in [Huang et al., 2007], we can always find a rank-one solution to the relaxed problem (13) so its principal eigenvector is the optimal solution to the original problem. For the cases where the problem has a solution with rank higher than one, several randomization techniques have been proposed which provide a good approximation to the rank- one SDP problem [Sidiropoulos et al., 2006].

Simulations

We provide some simulation results to show the performance of the proposed scheme in relay networks. We set all noise powers $\sigma_n^2 = \sigma_v^2 = 0dB$ and the transmit power of transmitter is assumed to be equal to 0dB ($P_0 = 0dB$). Also, the channel coefficient f and g are generated as i.i.d complex Gaussian random variables with variances σ_f^2 and σ_g^2, respectively. All

simulation results are averaged over 500 independent channel realizations. All MATLAB simulations are run on a PC with RAM=4GB, Processor: Intel (R) Core ™ i5-2400 CPU @ 3.10GHZ, System Type: 64bit.

In the first experiment, three methods to maximize SNR criterion are compared in the case of one source-destination with the help of 20 relays (R=20) and $\sigma_f^2 = \sigma_g^2 = 10dB$. As depicted in Fig. 2, the third algorithm outperforms the other two algorithms. So, for the next simulations, we are focused just on the third algorithm.

Fig. 3 illustrates the maximum achievable SNR versus the maximum allowable total relay transmit power for different number of relays. As shown in this figure, while we increase the number of relays, the maximum achievable SNR will be increased, because more antennas provide more spatial diversity. But, it should be noted that the complexity of the algorithm increases. So, we must make a tradeoff between the performance and complexity based on the system requirements and the available resources.

The remarkable result from Fig. 3. is that after a reasonably higher value of R, the performance improvement is not noticeable. It is due to this fact that increasing the number of relay antennas amplify not only the desired signal but also the noises.

In Fig. 4. and Fig. 5, the effects of the quality of uplink and downlink channels are considered when R=20. Notice that a larger variance of channel coefficients indicates a better channel [Goldsmith et al., 2004]. In Fig. 4, $\sigma_g^2 = 10dB$ and in Fig. 5, $\sigma_f^2 = 10dB$. As shown, SNR increases as the quality of the channels improves.

Conclusions

A system consisting of one source-destination was considered which communicated with each other just by the help of multi single antenna AF relays. We proposed three algorithms which maximize the SNR while the total transmit power in relays should be less than a certain level. In simulation results, it was obvious that when the number of relay nodes increased, the higher SNR can be achieved. In addition, simulation results show that if the uplink and downlink channels have a better quality, the achieved SNR can be improved.

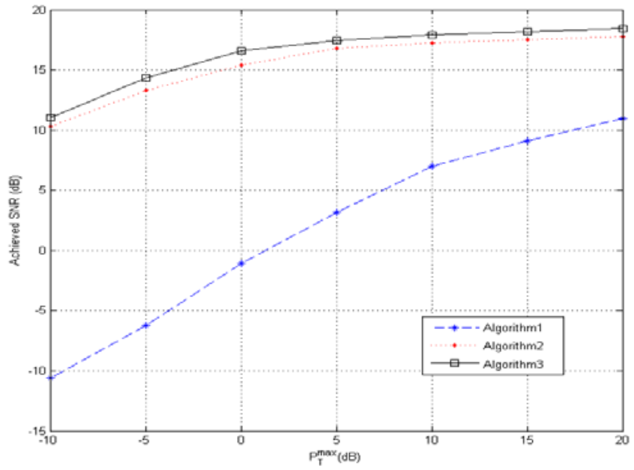

FIG. 2 AVERAGE MAXIMUM ACHIEVED SNR VERSUS RELAY TRANSMIT POWER FOR THE THREE PROPOSED ALGORITHMS

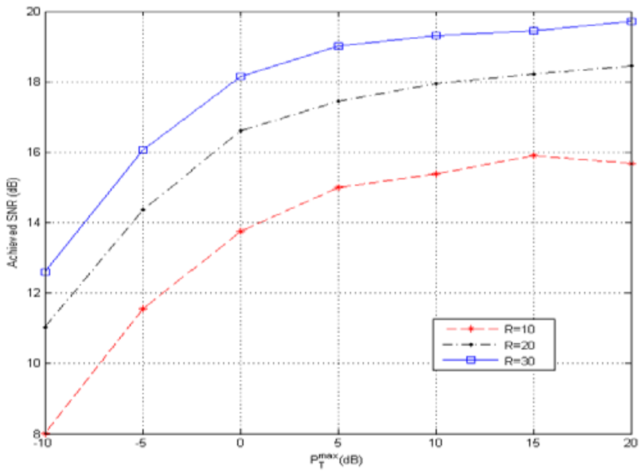

FIG. 3 AVERAGE MAXIMUM ACHIEVED SNR VERSUS RELAY TRANSMIT POWER FOR DIFFERENT NUMBER OF RELAYS

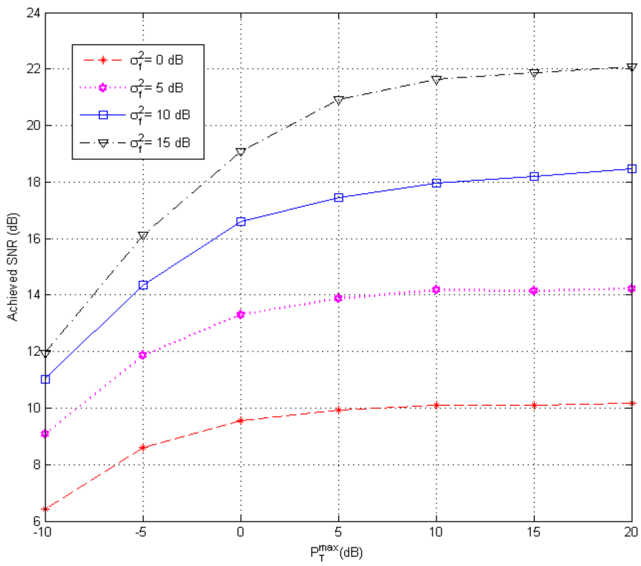

FIG. 4 AVERAGE MAXIMUM ACHIEVED SNR VERSUS RELAY TRANSMIT POWER FOR DIFFERENT VALUES OF σ_f^2

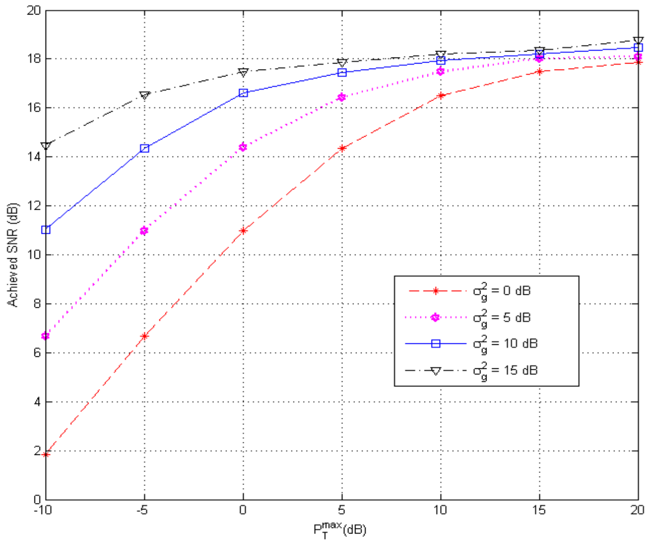

FIG. 5 AVERAGE MAXIMUM ACHIEVED SNR VERSUS RELAY TRANSMIT POWER FOR DIFFERENT VALUES OF σ_g^2

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